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## LETTER TO THE EDITOR

# Absence of localization in disordered systems with local correlation

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**Abstract.** In an early work by Dunlap *et al* it was conjectured, using a matrix-transfer approach, that diagonal-disordered systems with local correlation (dimer-models) exhibit an absence of Anderson localization. In this letter we find explicitly these delocalized states, and energies, using a suitable unitary transformation and the symmetries related to the model. Moreover, we expand these states around the critical energy and find a new  $1/\sqrt{E - E_c}$  divergence.

The problem of one-dimensional disordered systems, has currently gained increased interest. The localization of all states in these systems has been widely accepted, since the pioneering work of Anderson [1]. But recently several mechanisms to delocalize those systems have been discovered. In two dimensions the magnetic field delocalizes states near the Landau levels in the quantum Hall effect. In one dimension polymers like polyaniline, when doped appropriately, exhibit very high conductivities [2].

One of the simplest examples of delocalization due to local correlations, is the off-diagonal discrete disordered system [3]. It corresponds to a biatomic tight-binding alloy model, where the disorder is caused by altered spacings. The associated Schrödinger equation is

$$D_{l+1}\psi_{l+1} + D_l\psi_{l-1} = E\psi_l \quad D_{2l+1} = D_{2l} \quad (1)$$

where  $D_{2l}$  are random independent quantities. Equation (1) has a delocalized state for  $E = 0$  corresponding to  $\psi_{2l} = 0$  and  $\psi_{2l+1} = (i)^l$  whereby the localization length diverges as  $1/E^2$ . This is in contrast with the model without correlations, where the localization length diverges in the band centre, but the envelope of the wavefunction behaves as  $e^{-\sqrt{l}}$  [4]. In our case the envelope is independent of  $l$ . Therefore these states are extended in the same sense as Bloch wavefunctions. For continuous Kronnig–Penny disordered models, delocalization due to correlations have been noticed [6].

In this letter we study the so-called disordered diagonal dimer model corresponding to a one-dimensional tight-binding binary alloy, where the random site energies are assigned pairwise. The model was originally studied in [5] by using a transfer-matrix method. As Dunlap *et al* [5] noticed, the number of extended states, for a finite system of size  $N$  is proportional to  $\sqrt{N}$ . We work out the explicit extended wavefunction using symmetry properties. We then perturb this wavefunction around the critical energy and obtain a new  $1/\sqrt{E}$ -type divergence for the localization length, at the outer edges.

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Let us consider the disordered-diagonal tight-binding model described by the following Hamiltonian:

$$H = V_l |l\rangle \langle l| + |l\rangle \langle l+1| + |l\rangle \langle l-1| \quad (2)$$

where  $|l\rangle$  can be regarded as an atomic-like orbital centred at site  $l \in Z$ .  $V_l$  is the random potential taking the values  $\pm V$  (binary-alloy) and satisfying the dimer condition of local correlation  $V_{2l-1} = V_{2l}$  or, in terms of a probability distribution,

$$P(V_{2l}, V_{2l-1}) = \frac{1}{2} \delta(V_{2l} - V_{2l-1}) [\delta(V_{2l} - V) + \delta(V_{2l} + V)] \quad (3)$$

which shows explicitly that correlations between nearest sites exist.

For practical purposes it is convenient to define the unitary transformation

$$U|l\rangle = e^{i\phi l} |l\rangle \quad \cos \phi = V. \quad (4)$$

The corresponding transformed Hamiltonian  $H_U$  can be written as

$$H_U = V_l |l\rangle \langle l| + e^{-i\phi} |l\rangle \langle l+1| + e^{i\phi} |l\rangle \langle l-1|. \quad (5)$$

To solve the Schrödinger equation  $H_U |\psi\rangle = E |\psi\rangle$ , we consider the ansatz

$$|\psi\rangle = \sum \kappa_l (|2l\rangle + |2l+1\rangle) \quad (6)$$

which is suggested by the symmetries of the model. We obtain for  $\kappa_l$

$$\kappa_{l+1} V_{2l+2} + \kappa_l e^{i\phi} + \kappa_{l+1} e^{-i\phi} = E \kappa_{l+1} \quad (7)$$

$$\kappa_l V_{2l+1} + \kappa_l e^{i\phi} + \kappa_{l+1} e^{-i\phi} = E \kappa_l. \quad (8)$$

Using the dimer condition  $V_{2l+1} = V_{2l+2}$ , we derive the following condition on the energy  $E$ :

$$(E - V_{2l+2} - e^{i\phi})(E - V_{2l+2} - e^{-i\phi}) = 1 \quad (9)$$

which is satisfied for  $E = V$ . We point out that the relation  $V_{2l+2}^2 = V^2 = \cos^2 \phi$  was used. Moreover, when (9) is verified, which is the case for  $E = V$ , we obtain from (7) and (8) that  $|\kappa_{l+1}| = |\kappa_l|$ . Therefore we can conclude that this state at  $E = V$  is extended, namely,  $E = V$  corresponds to one delocalized state. It can be written explicitly from (8) as

$$\kappa_{l+1} = (V - V_{2l+2} - e^{i\phi}) e^{i\phi} \kappa_l. \quad (10)$$

In analogy, we can show the existence of another delocalized state for  $E = -V$ . This can be achieved by considering states of the form

$$|\psi\rangle = \sum_l \kappa_l (|2l\rangle - |2l+1\rangle) \quad (11)$$

and the corresponding coefficients are given by

$$\kappa_{l+1} = (V + V_{2l+2} - e^{i\phi}) e^{i\phi} \kappa_l. \quad (12)$$

In this way, we show that the states corresponding to  $E = \pm V$  are delocalized. The condition  $V \leq 1$  is necessary in order to validate the transformation (4). This condition on the random binary-alloy potential is also obtained in [5].

We will now restrict ourselves to the edge, i.e.  $V = 1$ , and, furthermore,  $V_{2l+1} = V_{2l+2} = W_{l+1}$ . In this case (11) takes the simple form

$$|\psi\rangle = \sum_l \psi_l |l\rangle \quad \psi_{2l+1} = \psi_{2l} = \prod_j^l (-W_j). \quad (13)$$

Rewriting (5) for the energy  $E = 1 + \epsilon^2$  we obtain

$$\psi_{2l+1} + \psi_{2l-1} = (1 - W_l + \epsilon^2)\psi_{2l} \quad \psi_{2l+2} + \psi_{2l} = (1 - W_{l+1} + \epsilon^2)\psi_{2l+1}. \quad (14)$$

We make the following Ansatz  $\psi_l \rightarrow \gamma_l \xi_l$  where

$$\xi_l = 1 + \alpha_l \epsilon + \beta_l \epsilon^2 \quad \gamma_{2l} = W_l \quad \text{and} \quad \gamma_{2l+1} = 1. \quad (15)$$

Considering  $\epsilon$  small, of the order  $1/N$  where  $N$  is the size of the system, we restrict ourselves to the second order, yielding

$$1/\xi = 1 - \alpha_l + (\alpha_l^2 - \beta_l)\epsilon^2. \quad (16)$$

Inserting (15) and (16) in (14) we obtain, by equating each order of  $\epsilon$ ,

$$W_l \alpha_{2l} = -\alpha_{2l+1} \quad W_{l+1} \alpha_{2l+2} = -\alpha_{2l+1} \quad (17)$$

and

$$\beta_{2l+1} = W_l \alpha_{2l+1}^2 - W_l \beta_{2l} + 1 \quad \beta_{2l+1} = \alpha_{2l+1}^2 - W_{l+1} \beta_{2l+2} - 1. \quad (18)$$

Notice that  $W_l = \pm 1$ . Moreover, we fix the initial condition as  $\psi_0 = 1$  and  $W_1 = -1$ . From (17) we obtain

$$W_l \alpha_{2l} = -\alpha \quad \alpha_{2l+1} = \alpha \quad (19)$$

where  $\alpha = \alpha_2$  and  $\beta = \beta_2$ . In order to simplify (18) we fix  $\alpha^2 = 2$ , and obtain

$$W_{l+1} \beta_{2l+2} = -\beta - 2 \sum_j^l W_j \quad \beta_{2l+1} = 1 + \beta + 2 \sum_j^l W_j. \quad (20)$$

If we take the probability  $p$  or  $(1-p)$  of the occurrence of either  $W_l = +1$  or  $W_l = -1$  respectively, being close to  $\frac{1}{2}$ , then  $|\beta_l| \ll N$ , so restricting us to the first order in  $\epsilon$  for the expression of  $\psi_l$ . Inserting (19) and (20) in (15) yields

$$\frac{\psi_{2l}}{\psi_{2l-1}} = -W_l + \alpha \cdot \epsilon \quad \frac{\psi_{2l+1}}{\psi_{2l}} = 1 + \alpha \cdot \epsilon. \quad (21)$$

We can now evaluate the complex Lyapounov exponent  $\gamma$  defined as [7]

$$\gamma = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{l=1}^N \ln \left( \frac{\psi_{l+1}}{\psi_l} \right) \quad (22)$$

where the complex logarithm is defined such that  $\ln(-x) = i\pi + \ln(x)$ , if  $x$  is real and positive. The Lyapounov exponent, or the inverse localization length  $L_c^{-1}$ , is the real part of  $\gamma$  and the integrated density of states  $Z$  is the imaginary part. For the inverse localization length we obtain

$$L_c^{-1} = p\alpha \cdot \epsilon \quad (23)$$

and for the integrated density of states

$$Z = \frac{1}{2}(1 - p)(\epsilon + 1). \quad (24)$$

If we equate the localization length with the size  $N$  of the system, we can estimate the number of extended states, which will be of the order of unity and independent of  $N$ . Therefore none of these states would contribute to transport properties of the system.

The result is the same if we consider  $V = -1$ . The general result for the localization length is hence

$$L_c \sim \frac{1}{\sqrt{E-1}} \quad \text{for } E > 1 \quad (25)$$

and

$$L_c \sim \frac{1}{\sqrt{-(E+1)}} \quad \text{for } E < -1. \quad (26)$$

In conclusion, starting from the diagonal disordered dimer model, we have found explicit extended states (10) and (12), with energies  $E = \pm V$ . Delocalized states for this model were obtained in [5] using a transfer-matrix method. They evaluated the reflection coefficient  $|R|^2$  of a single dimer impurity. When  $R$  is zero, adding up any number of dimer impurities does not alter the extended states. Moreover, they constructed a wavefunction, by adding a different phase whenever a dimer impurity occurs. Our approach is very different. We start from an *overall* disordered dimer model and use the symmetry properties of the model, and a unitary transformation, to end up with a solution for the wavefunction. It is equivalent to the one obtained in [5], if the same configuration of the dimer impurities is used. The point is that the solution is very direct, and does not imply calculating reflection coefficients. It is, moreover, almost straightforward to obtain extended solutions for trimer and any-mer models. The problem in calculating the reflection coefficient, for just one dimer impurity, is that if it has a non-zero value one must add them up (in a non-trivial way) for each dimer impurity. For high concentrations of impurities this may become very difficult. In our case, by perturbing the wavefunction, we immediately have a result for high concentrations of dimer impurities.

To understand the transport properties of these systems, it is important to know the behaviour around the critical energy. For the case  $V < 1$ , there is a superdiffusive behaviour at the critical energy [5], leading to a  $1/E^2$  singularity of the localization length. This dependence doesn't change if instead of dimers, any-mers are used [8]. Even in the non-diagonal disordered dimer model, the same singularity holds [3]. Dunlap *et al* [5] first argued that the number of extended states is proportional to  $\sqrt{N}$ . Bovier [7] confirmed this result by calculating the integrated density of states, using the invariant measure formalism. He also showed that when  $V = 1$  and  $|E| \leq 1$ , the same proportionality in  $\sqrt{N}$  holds, although the localization length diverges as  $1/E$ , in contrast to the case  $|E| > 1$ , where it diverges as  $1/\sqrt{E}$ .

In order to estimate the transport properties, we have to keep in mind that the real energy is with the opposite sign, as the negative sign of the Schrödinger operator has been omitted in (2). Therefore the state with  $E = 1$  corresponds to a lower level than the one with  $E = -1$ . Moreover, we restrict ourselves to the case  $V = 1$ . Therefore, when the Fermi energy  $E_F > 1$ , no states will contribute to the conductivity, as follows from (24) and (25). However, for  $E_F \leq 1$  there will be a non-zero conductivity as the number of extended states is proportional to  $\sqrt{N}$ . This clearly shows that the system exhibits a *sharp* mobility edge transition.

Although notable progress has been made in understanding delocalization effects, there still remain important open questions. Up to now no general criteria are known leading to these effects. In the two-dimensional dimer model explicit extended Eigenfunctions can be found [9], but a general study of the effect of local correlations is still missing.

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